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CALCULATION OF TRANSIENT RESPONSE FROM TIME-HARMONIC SPECTRUM

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J H James

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CALCULATION OF TRANSIENT RESPONSE FROM TIME-HARMONIC SPECTRUM

By

J H James

Summary

Theory behind a Fortran program (not listed) which facilitates computations of transient response is described. Discrete values of a time-harmonic spectrum are used, via quadratic interpolation and weighting by the transform of a selected time function, to provide samples for transient response calculations done by numerical evaluation of a Fourier integral using the fast Fourier transform (FFT) algorithm. Examples of motion, radiation and scattering transient responses of simple dynamical systems demonstrate that the program is a valuable research tool. This report is an update of previous work.

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CALCULATION OF TRANSIENT RESPONSE FROM TIME-HARMONIC SPECTRUM

By J H James

1. INTRODUCTION

Transient response calculations frequently reveal physical features which are difficult to extract from the corresponding time-harmonic spectra, see for example James (1). In order to facilitate transient sound radiation and scattering calculations a Fortran program (2) has been written to produce transient time histories from user supplied time-harmonic spectra together with user selected time excitations. Numerical examples demonstrated that the program is a valuable tool for transient far-field radiation and scattering computations. It was noted that while the program had been designed for calculations of transient far-field pressure the amendments required for it to be usable for other transient response problems were trivial.

These amendments have now been made and it is the purpose of this report to provide an update to the theoretical work and numerical examples. Much of the text also appears in the previous version (2) which has a limited availability.

2. TRANSIENT RESPONSE

a. General Response

The solution of time-harmonic response problems at time t and field-point x can be expressed in the form

$$p(x,t) = S(x,\omega)\exp(-i\omega t) \tag{2.1}$$

where $p(x,t)$ is the time-harmonic response, which is the response due to $\exp(-i\omega t)$ time variation of the excitation; $\omega = 2\pi f$ is the radian frequency of oscillation; $S(x,\omega)$ is the complex response spectrum. A diagram illustrating typical responses is shown in Figure 1.

The transient response for separable time variation $Q(t)$ of the excitation is simply

$$p(x,t) = \int_{-\infty}^{\infty} Q(\omega)S(x,\omega)\exp(-i\omega t)d\omega \tag{2.2}$$

where $Q(\omega)$ is the transform of $Q(t)$, viz

$$Q(\omega) = (1/2\pi)\int_0^T Q(t)\exp(i\omega t)dt \tag{2.3}$$

in which the excitation $Q(t)$ is assumed to vanish outside the time interval $t = 0,T$.

Equation (2.2) can be evaluated numerically from user supplied values of $S(x,\omega)$ and $Q(\omega)$. From a practical point of view it is better written as

$$p(x,t) = \int_{-\infty}^{\infty} Q(\omega)S(x,\omega)\exp(-i\omega(t-t_0))d\omega \tag{2.4}$$

where t_0 is the transient response delay factor, introduced in order to check for non-casual behaviour in the plotted time history; see Section 5 for numerical examples.

b. Far-Field Radiation

The solution of time-harmonic radiation/scattering problems in the far-field is usually expressed in the form

$$p(R,\theta,\phi,t) = A(\theta,\phi,\omega)[\exp(ikR)/R]\exp(-i\omega t) \qquad (2.5)$$

in which $p(R,\theta,\phi,t)$ is the time-harmonic pressure; (R,θ,ϕ) are spherical co-ordinates; $k = \omega/c$ is the acoustic wavenumber where c is the sound velocity in the fluid; $A(\theta,\phi,\omega)$ is the angular distribution of the radiation field. This equation is often evaluated at the reference distance of one metre, but many computer codes simply evaluate $A(\theta,\phi,\omega)$.

The transient pressure field for separable time variation Q(t) of the excitation is simply

$$p(R,\theta,\phi,t) = (1/R) \int_{-\infty}^{\infty} Q(\omega)A(\theta,\phi,\omega)\exp[ik(R-ct)]d\omega \qquad (2.6)$$

which is the same as equation (2.4) provided $S(x,\omega) = A(\theta,\phi,\omega)/R$, and $t_0 = R/c$.

For far-field transient radiation the standard reference distance of one metre is chosen for the divisor, R. Note all responses are at relative times because the far-field is really at infinity. This may cause conceptual problems because in certain circumstances causality appears to be violated, which is perhaps not the case. For instance, there may be the expectation that the phase term (R-ct) will cause the transient pressure to be zero before time t = R/c, but this may not be the case as supersonic elastic waves travelling on the surface of the radiator could cause radiation to leave a remote point and arrive at the observation point before the direct travel time, see for example James (1). Because the transient far-field pressure is invariant with respect to R-ct, it is convenient to use this term simply as a device for shifting the excitation in time. Thus, set $t_0 = R_p/c$ where R_p is large enough, greater than a typical body dimension say, to include responses originating before t = 0. This scheme is an especially valuable facility because the integration algorithm, described in Section 4, produces responses at equidistant times, starting at relative time t = 0, which means that responses originating before t = 0 are lost, unless a sufficiently large value of R_p is chosen.

c. Far-Field Scattering

For a time varying plane wave, incident at the angles (θ_i,ϕ_i), the insonification is of the form

$$p_i(x,y,z,t) = \int_{-\infty}^{\infty} P_i(\omega)\exp(-ik\alpha_i)\exp(-i\omega t)d\omega \qquad (2.7)$$

where

$$\alpha_i = x.\sin(\theta_i)\cos(\phi_i)+y.\sin(\theta_i)\sin(\phi_i)+z.\cos(\theta_i) \tag{2.8}$$

and

$$p_i(x,y,z,t) = P_i\exp(-ik\alpha_i)\exp(-i\omega t) \tag{2.9}$$

is the incident pressure excitation used in the time-harmonic problem.

Let the scatterer be surrounded by a sphere of radius L. The point on this sphere first met by the incident plane wave is $x_s = L.\sin(\theta_i)\cos(\phi_i)$, $y_s = L.\sin(\theta_i)\sin(\phi_i)$ and $z_s = L.\cos(\theta_i)$. At this point the incident pressure wave is obtained from equation (2.7) as

$$p_i(x_s,y_s,z_s,t) = \int_{-\infty}^{\infty} P_i(\omega)\exp(-ikL)\exp(-i\omega t)d\omega \tag{2.10}$$

Let this incident wave be the real valued pressure pulse of finite duration,

$$\begin{aligned} p_i(x_s,y_s,z_s,t) &= Q(t), \text{ for } 0<t<T \\ &= 0, \text{ for } t<0 \text{ and } t>T \end{aligned} \tag{2.11}$$

to give, on taking the inverse Fourier transform,

$$\begin{aligned} P_i(\omega) &= [\exp(ikL)/2\pi]\int_0^T Q(t)\exp(i\omega t)dt \\ &= \exp(ikL)Q(\omega) \end{aligned} \tag{2.12}$$

The transient pressure scattered to the far-field is now synthesized from the time-harmonic pressure equation (2.5) as the integral

$$p(R,\theta,\phi,t) = (1/R)\int_{-\infty}^{\infty} Q(\omega)A(\theta,\phi,\omega)\exp[ik(L+R-ct)]d\omega \tag{2.13}$$

which is the same as equation (2.4) provided $S(x,\omega) = A(\theta,\phi,\omega)/R$ and $t_0 = (L+R)/c$. As before set $t_0 = R_p/c$ where R_p is chosen large enough, > 3L say, to include responses originating before t = 0.

3. TRANSFORMS OF EXCITATIONS IN TIME

a. Impulse

The impulse applied at t = 0 is defined as

$$Q(t) = P\delta(t) \tag{3.1}$$

where P is the magnitude of the impulse. Its transform is obtained from equation (2.3) as

$$Q(\omega) = P/2\pi \tag{3.2}$$

b. Heaviside

The Heaviside excitation applied at t = 0 is defined as

$$Q(t) = F_0, \text{ for } 0<t<\infty \tag{3.3}$$

$$Q(t) = 0, \text{ for } t<0$$

Its principal value transform is

$$Q(\omega) = F_0 i/(2\pi\omega) \tag{3.4}$$

c. Sine Wave

The sine wave which is switched on at t = 0 and switched off at t = T is defined as

$$Q(t) = F_0 \sin(\omega_0 t), \text{ for } 0<t<T \tag{3.5}$$

$$Q(t) = 0, \text{ for } t<0 \text{ and } t>T$$

the time for one cycle being $1/f_0 = 2\ /\omega_0$. Its transform is

$$Q(\omega) = (F_0/2\pi)[\exp(i\omega T)\{i\omega\sin(\omega_0 T)-\omega_0\cos(\omega_0 T)\}+\omega_0](\omega_0{}^2-\omega^2) \tag{3.6}$$

d. Squared Sine Wave

The squared sine wave which is switched on at t = 0 and off at t = T is defined as

$$Q(t) = F_0 \sin^2(\omega_0 t), \text{ for } 0<t<T \tag{3.7}$$

$$Q(t) = 0, \text{ for } t<0 \text{ and } t>T$$

the time for a half-cycle (one loop) being $1/2f_0 = \pi/\omega_0$. Its transform is

$$\begin{aligned} Q(\omega) &= (F_0/4\pi i\omega)[\exp(i\omega T)-1] \\ &\quad -(F_0/4\pi)[\exp(i\omega T)\{i\omega\cos(\omega_0 T)+2\omega_0\sin(2\omega_0 T)\}-i\omega]/(4\omega_0{}^2-\omega^2) \end{aligned} \tag{3.8}$$

e. Attenuated Sine Wave

The attenuated sine wave which is switched on at t = 0 and off at t = T is defined as

$$Q(t) = F_0 \exp(-at)\sin(\omega_0 t), \text{ for } 0<t<T \tag{3.9}$$

$$Q(t) = 0, \text{ for } t<0 \text{ and } t>T$$

the time for one cycle being $1/f_0 = 2\pi/\omega_0$. Its transform is

$$Q(\omega) = (F_0/2\pi)[\exp(i\lambda T)\{i\lambda\sin(\omega_0 T)-\omega_0\cos(\omega_0 T)\}+\omega_0]/(\omega_0^2-\lambda^2)$$

in which $\lambda = \omega+ia$, with $|a|>0$. (3.10)

f. Triangular Pulse

The triangular pulse which is switched on at $t = 0$ and off at $t = T$ is defined as

$$Q(t) = (2F_0/T)t, \text{ for } t = 0<t<T/2$$

$$Q(t) = -(2F_0/T)t+2F_0, \text{ for } T/2<t<T \qquad (3.11)$$

$$Q(t) = 0, \text{ for } t<0 \text{ and } t>T$$

Its transform is

$$Q(\omega) = (-2F_0/2\pi\omega^2 T)[1-\exp(i\omega T/2)]^2 \qquad (3.12)$$

g. Rectangular Pulse

The square pulse which is switched on at $t = 0$ and off at $t = T$ is defined as

$$Q(t) = F_0, \text{ for } 0<t<T \qquad (3.13)$$

$$Q(t) = 0, \text{ for } t<0 \text{ and } t>T$$

Its transform is

$$Q(\omega) = (F_0/2\pi i\omega)[\exp(i\omega T)-1] \qquad (3.14)$$

This pulse may be of more practical value than the impulse and the Heaviside excitations in a. and b. above.

h. Gaussian AM Sine Wave

The Gaussian amplitude modulated sine wave which is switched on at $t = 0$ and off at $t = T$ is defined as

$$Q(t) = F_0\exp(-at^2)\sin(\omega_0 t), \text{ for } 0<t<T$$

$$Q(t) = 0, \text{ for } t<0 \text{ and } t>T \qquad (3.15)$$

Its transform has no simple closed form solution, thus it is obtained by numerical integration of equation (2.3).

i. Linear Sweep FM Sine Wave

The linear swept, frequency modulated, sine wave which is switched on at $t = 0$ and off at $t = T$ is defined as

$$Q(t) = F_0 \sin(at+bt^2), \text{ for } 0<t<T$$
$$Q(t) = 0, \text{ for } t<0 \text{ and } t>T \tag{3.16}$$

in which $a+2bt$, the differential of the argument, is defined as the instantaneous frequency. Let ω_0 and ω_T be the lower and upper frequencies of the sweep, then $a = \omega_0$ and $b = (\omega_T-\omega_0)/2T$. The transform of this excitation has no simple closed form solution so it is also obtained by numerical integration.

j. User Defined Excitation

The Fourier transform of a user selected transient excitation, for example the amplitude and frequency modulated sine wave $a(t)\sin(b(t))$, can be obtained by numerical quadrature of equation (2.3). This user pulse is trivially inserted into the computer program (mentioned in Section 5) by following the coding of excitations h. and i., provided the pulse discontinuities, if any, are at the limits (0,T) of the integral. The coding evaluates integral transforms by Simpson's rule applied to M equally spaced values of the excitation. When the excitation has interior discontinuities the integration range should be split into separate intervals, bounded by the discontinuities.

4. FFT EVALUATION OF FOURIER INTEGRAL

The discretized version of equation (2.4), which is of the form

$$F(t) = \int_{-\infty}^{+\infty} H(\omega)\exp(-i\omega t)d\omega \tag{4.1}$$

with $H(\omega)=Q(\omega)S(x,\omega)\exp(i\omega t_0)$ has been given by Cooley et al. (3) as

$$F(t_j) = \delta\omega\Sigma_{k=0}^{N-1} G_k\exp(-2\pi ijk/N), \quad t_j=j\delta t, \; j=0, N-1 \tag{4.2}$$

with

$$G_k = \Sigma_{p=-\infty}^{\infty} H(k\delta\omega+Np\delta\omega) \tag{4.3}$$

N is an integer power of two in order to be compatible with the fast Fourier transform algorithm (4), for which the sampling intervals in frequency and in time, $\delta\omega$ and δt, are inextricably linked by the equation

$$\delta\omega = 2\pi/N\delta t \tag{4.4}$$

Thus, the infinite range of integration has been divided into intervals of length $N\delta\omega$ to allow construction of a quadrature formula which requires a single application of the FFT algorithm to evaluate the integral at

N distinct values of the time t. The time series is periodic, viz $F(t_j) = F(t_{j+N})$, but provided the true time function decays sufficiently rapidly in the interval $t = 0, N\delta t$ it may adequately be approximated by discrete samples derived from the FFT quadrature formula.

Some trial and error is usually necessary before the constants required for numerical integration are finally selected. The following suggestions may be found useful:

a. First, select the sampling interval in frequency, $\delta\omega$, and the upper limit of integration, ω_m, either by a guess or from prior knowledge of the spectrum $H(\omega)$. Choose N (power of two) and δt in accordance with equation (4.4).

b. Stop the summation, in equation (4.3), at the integer value (minimum 1) p_m which is greater or equal to $\omega_m/N\delta\omega$. Note that this may re-define the upper limit of integration as $\omega_m = p_m N\delta\omega$.

c. Use of the relation $H(-\omega)=H^*(\omega)$, reflecting a real valued function F(t), will halve computation times.

d. Use not more than the first N/2, say, values of the time function for plotting, as aliasing might have affected the values at the end of the range.

e. If the time history plot is obviously in error, being significantly non-casual for instance, then perhaps $\delta\omega$ is too large or ω_m is too small.

f. When the spectrum $H(\omega)$ is computationally expensive it can be extended with zeros, in order to enhance the resolution in time. Also the number of costly computations of $H(\omega)$ can be kept to a minimum if an interpolation scheme is used.

5. NUMERICAL EXAMPLES

a. General

A FORTRAN program has been written to evaluate the Fourier integral

$$p(x,t) = \int_{-\infty}^{\infty} Q(\omega)S(x,\omega)\exp(-i\omega(t-t_0))d\omega$$

numerically from a user supplied array of the discretized spectrum $S(x,\omega)$, at positive values of ω, and a user selected excitation Q(t). This user supplied array need not be at equal frequency intervals as quadratic interpolation is used to find values required for numerical integration by FFT. Interpolation is especially important when time-harmonic responses are costly. For instance, if a spectrum obtained by the finite element method has sharp peaks it may be desirable to form a composite spectrum from a crude resolution over the entire frequency range combined with several highly resolved regions where peaks are dominant. The program automatically pads the user supplied spectrum with zeros if its highest frequency is less than the maximum frequency $\omega_m=p_m N\delta\omega$ in the FFT quadrature formula. This useful feature allows selection of a time resolution which is small enough to obtain smooth plots of the transient response, without the added expense of unnecessary high frequency computations of $S(\omega)$.

For the numerical examples the following table summarizes the sampling constants used, frequency values being in hertz:

Spectrum Constants					FFT Constants				
Fig	NF	FMIN	FMAX	f	p_m	N	t	f	f_m
2a	1001	0.25	1000.0	1.0	1	8192	0.000500	0.244	2000
2b	201	0.25	1000.0	5.0	1	8192	0.000500	0.244	8000
3	1001	0.25	1000.0	1.0	1	16384	0.000125	0.244	8000
4	1001	0.25	1000.0	1.0	1	16384	0.000125	0.244	2000
5	501	1.00	1000.0	2.0	1	2048	0.000250	1.953	4000
6	1001	10.0	1000.0	.99	1	8192	0.000200	0.610	5000

where for the time-harmonic spectrum constants, NF is the number of frequency values ranging from FMIN to FMAX with a resolution of δf; and for the FFT constants f_m is the upper frequency limit of integration. Padding is brought into effect in all examples because the upper frequency limit in the FFT constants is greater than the upper frequency limit in the spectrum constants. At the upper frequency limit of each spectrum the product $Q(\omega)S(\omega)$ is small enough to allow truncation of the Fourier integral without any significant loss of accuracy. Computational times for all examples are trivial, when compared to the times needed to evaluate the spectra $S(\omega)$.

When the material is steel the following SI units are used; Young's modulus 19.5×10^{10}, Poisson's ratio 0.29, shear modulus 7.558×10^{10} and density 7700: water has density 1000 and sound velocity 1500.

b. <u>Impulse Response of Mass-Spring</u>

In the first example the grounded, heavily damped, mass and spring system shown in Figure 2 is subject to a unit impulse force. The constants in SI units are mass 1000, stiffness 1.0×10^8 and hysteretic loss factor 0.1. The displacement response spectrum $S(\omega)$ was calculated by a program, called COUPLE (5), which evaluates the response of a complex dynamical system by combining dynamic stiffness matrices of simple dynamical systems. The spectrum $S(\omega)$ is smooth except in the region of the mass/spring resonant frequency at 50.3 Hz. The spectrum $Q(\omega)$ of the excitation is flat.

The impulse response is shown in Figure 2a, the time delay being selected to start the response at t = 80 ms. It is in excellent agreement with Milne (6) who shows mathematically and demonstrates numerically that the very small non-causal response just before t = 80 ms is due to the selection of hysteretic damping. A plot (not shown here) has also been obtained in which the damping is viscous rather than hysteretic: the non-causal response, as expected, is not present.

Figure 2b shows non-causal response behaviour introduced by under-sampling the response spectrum $S(\omega)$. In this case the frequency spacing in $S(\omega)$ is too large for accurate interpolation. The response is acceptable for practical computations, but it could be improved dramatically by inserting about four additional $S(\omega)$ values centred on the 50.3 Hz natural frequency.

c. Axial response of FM Excited Beam

In the second example the uniform beam shown in Figure 3 is excited, in the axial direction, by a linear FM sine wave force of unit amplitude whose frequency sweeps from 50 to 100 Hz in 20 ms; see excitation in Figure 1. The steel beam is of length 20, area cross-section 0.012996 and has an hysteretic loss-factor of 0.01. The acceleration response spectrum $S(\omega)$ at each of the beam tips was calculated by COUPLE. They vary rapidly, with prominent resonances at 126 Hz spacing. The spectrum $Q(\omega)$ of the excitation drops rapidly above 400 Hz.

The axial acceleration responses at the ends of the beam are shown in Figure 3, the time delay being selected to start the responses at t = 10 ms. The responses are substantially causal before t = 10 ms. The response of the left-hand-side tip in Figure 3a resembles the excitation initially, and then extended ringing is evident, the beating being due to interaction among the first three resonances. Compare with the response of the right-hand-side tip in Figure 3b: the plots are almost identical after relative time 40 ms, when allowance is made for a delay of 4 ms for the wave to reach the far end of the beam at the longitudinal wave speed of 5032 ms^{-1}; levels in the former plot being just a fraction higher because of the effect of damping.

d. Impact Response of Isolation System

In the third example the simple isolation system shown in Figure 4 is excited at its base by a unit triangular pulse force which is on for 10 ms. The machine is modelled as a steel Timoshenko beam in bending with length 1.0, area cross-section 0.012996, moment of area in bending 1.407×10^{-5}, hysteretic loss-factor 0.01 and shear correction factor 0.822. The isolators are modelled as simple springs, which do not transmit bending moments, of stiffness 0.333×10^{8} and loss-factor 0.01. The foundation is modelled as an infinite steel plate of thickness 0.01 and loss-factor 0.02. The vertical acceleration responses $S(\omega)$ were calculated by COUPLE. They vary slowly except for a prominent peak at 564 Hz which is due to the first bending resonance of the beam. There is no sharp mass-spring resonance because the machine is mounted on a low impedance, but highly resistive, foundation. The spectrum $Q(\omega)$ of the excitation drops smoothly to zero at 200 Hz and its first side lobe at 300 Hz is 27 dB down.

The vertical acceleration responses of the tips of the beam are shown in Figure 4, the time delay being selected to start the responses at t = 80 ms. The response of the left-hand-side tip in Figure 4a resembles the excitation, with small perturbations, for the first 10 ms and then extended ringing is evident. Compare with the response of the right-hand-side tip in Figure 4b: the initial response is of opposite phase while the extended ringing is of the same phase. These observations are consistent with the early response being dominated by the heave and pitch modes of the beam and the later response being due to ringing at the fundamental beam resonant frequency. Interaction through the water loaded plate is probably small.

e. Radiation and Scattering of Shell

The fourth example is of far-field radiation from and scattering by the thin elastic steel shell shown in Figure 5. The excitation is a transient force or plane wave, of unit amplitude, which has the form of a 300 Hz sine wave which is on for one cycle. Shell constants are radius 1.0, thickness 0.01 and hysteretic loss-factor 0.01. Radiation and scattering spectra $S(\omega)$ were calculated from closed-form theoretical expressions (7,8). They vary rapidly in the mid-frequency regime due to resonance effects. The spectrum $Q(\omega)$ of the excitation has a broad lobe near to 250 Hz and its first side lobe near 725 Hz is 18 dB down.

Figure 5 shows plots of the far-field pressure transients. Small ripples at times before 15 ms are caused by premature truncation of the spectra $S(\omega)$. For the radiation transient the initial pressure has the same phase as the excitation, probably due to the force being applied directly to the water. For the scattering transient the initial pressure has the opposite phase, probably due to the incident wave initially seeing a pressure release shell. Extended ringing caused by resonances is very marked when it is recalled that the excitation lasts for only 3.3 ms.

These plots also appear in the earlier publication (2) and there they are accompanied by plots of spectra $S(\omega)$ derived from both closed-form and finite element solutions.

f. Radiation from Water-Filled Pipe

The final example is of far-field airborne sound radiation from the water-filled steel pipe shown in Figure 6. The excitation is a transient point source, of unit amplitude, which has the form of a 250 Hz sine wave which is switched on for either one cycle or forty cycles. The pipe has radius 0.10, thickness 0.005 and hysteretic loss-factor 0.01. The radiation spectrum $S(\omega)$ was calculated from closed-form theoretical expressions (9). It varies smoothly except for sharp resonances at 225 and 685 Hz. The spectrum $Q(\omega)$ of the excitation has its main lobe near 250 Hz. Figure 6 shows plots of the far-field pressure transients. Small ripples at times before the main response starts are caused by premature truncation of the spectra $S(\omega)$. In Figure 6a the excitation is a single cycle sine wave; the initial response closely resembles the excitation, and the later response is dominated by slowly decaying ringing of the 225 Hz resonance. In Figure 6b the rapid transition to a near steady state is shown. The beating, slowly decreasing with time around the time-harmonic level of $\pm$ 0.44, is caused by interaction between the continuous excitation and the decaying resonance transient. When the excitation is switched off after 40 cycles the amplitude drops rapidly, leaving decaying ringing due to the resonance transient.

6. CONCLUDING REMARKS

Transient response (motion, radiation, scattering, etc) of a dynamical system can be expressed as a Fourier transform over frequency of the time-harmonic pressure weighted by the transform of the time varying part of the excitation, assumed separable in space and time. This transform is evaluated numerically by an application of the FFT algorithm. The main features that have arisen from the work reported herein are as follows:

a. A FORTRAN program which evaluates a transient response from a user supplied array of time-harmonic responses, which need not be at equidistant frequency intervals as interpolation is used to obtain the values required for numerical integration. The array of pressures may be extended automatically with zeros in order to improve resolution in time. Computer time is trivial.

b. A number of basic transient excitations which can be selected by the user of the program. It is not difficult to insert additional excitations into the computer program, even if closed-form expressions are not available for their transforms, as a scheme for numerical integration is built into the program.

c. Numerical examples of transient responses which provide guidelines for the program's use and also demonstrate that it is a powerful research tool.

Computations of transient response of linear systems are usually done either by 'time marching' on the original time-dependent differential equations (or matrices) or by Fourier transform of the time-harmonic solutions, as herein. For general response calculations the latter method would appear to have an advantage not only because computer programs for time-harmonic problems are widely available but also because the appearance of non-causal responses are an indication of errors, usually due to under-sampling or premature truncation of the time-harmonic spectrum $S(\omega)$. Time marching methods implicitly assume causality.

Future extensions to the program should include a limited capability for filtering the transient response, as is usually the case in physical measurement. A causal filter, with frequency response spectrum $F(\omega)$, is easily inserted in the computer program as the transient response is now the Fourier transform of $F(\omega)Q(\omega)S(\omega)$.

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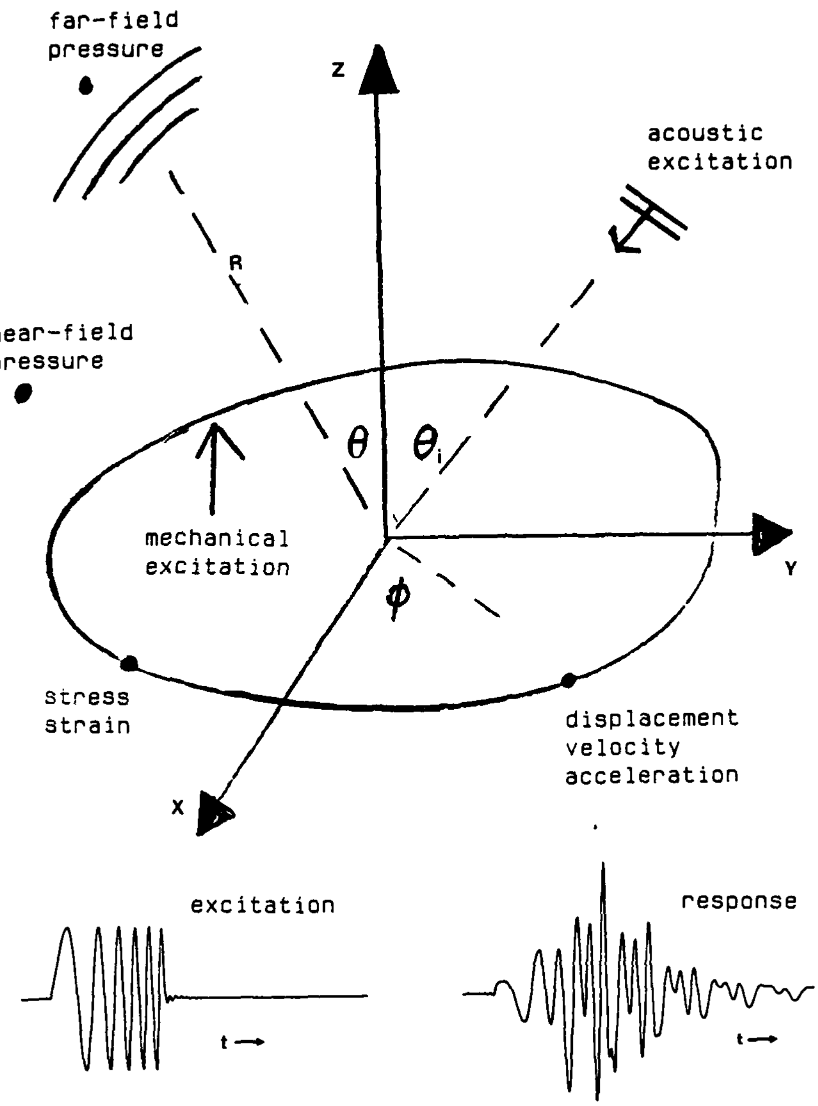


FIG. 1 Typical transient responses of dynamical system

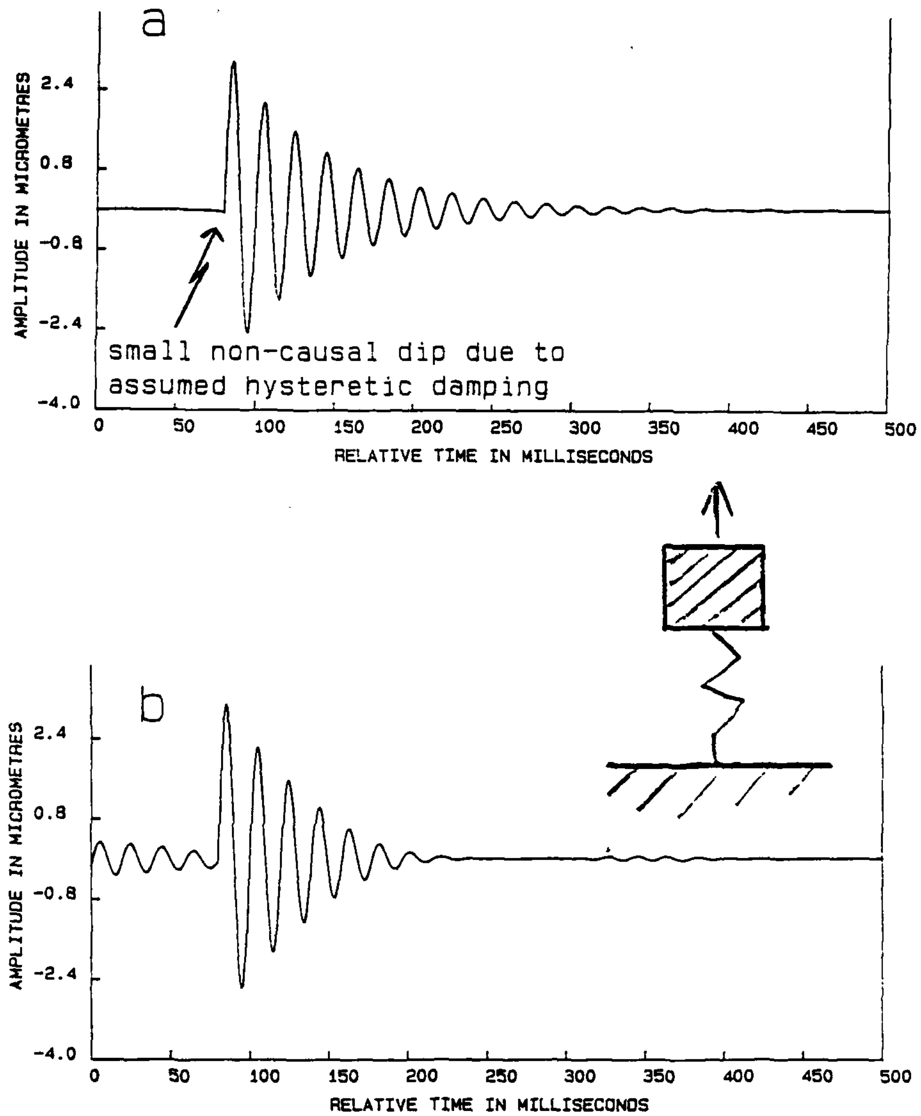
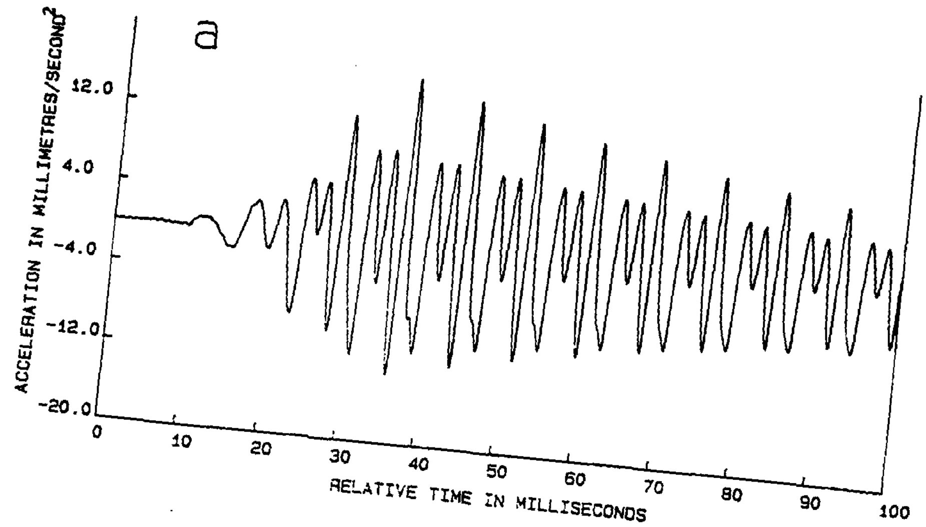


FIG. 2 Impulse response of damped mass-spring system: (a) overdetermined spectrum sampling gives excellent results and (b) underdetermined spectrum sampling gives a non-causal response.



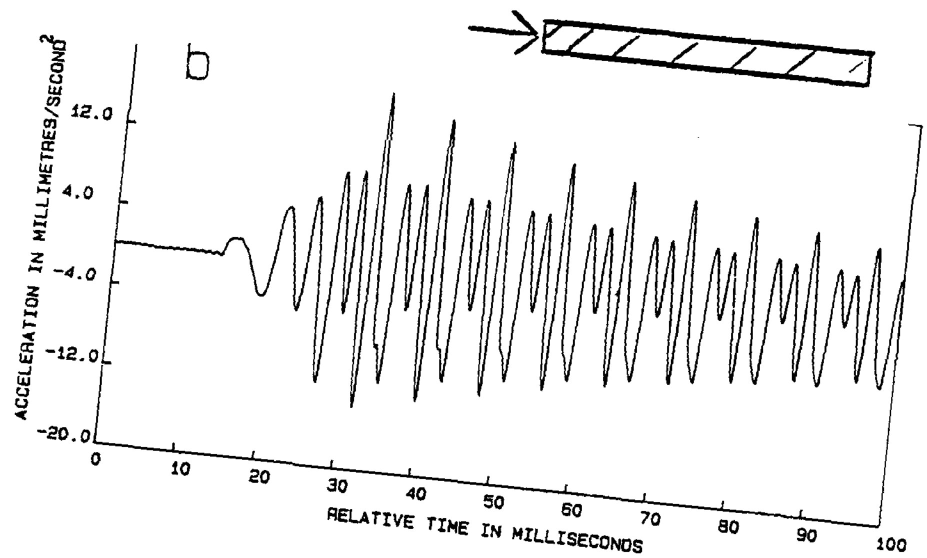


FIG. 3 Axial acceleration response of beam excited by sine wave sweeping from 50 to 500Hz in 20ms: (a) at near beam tip and (b) at far beam tip.

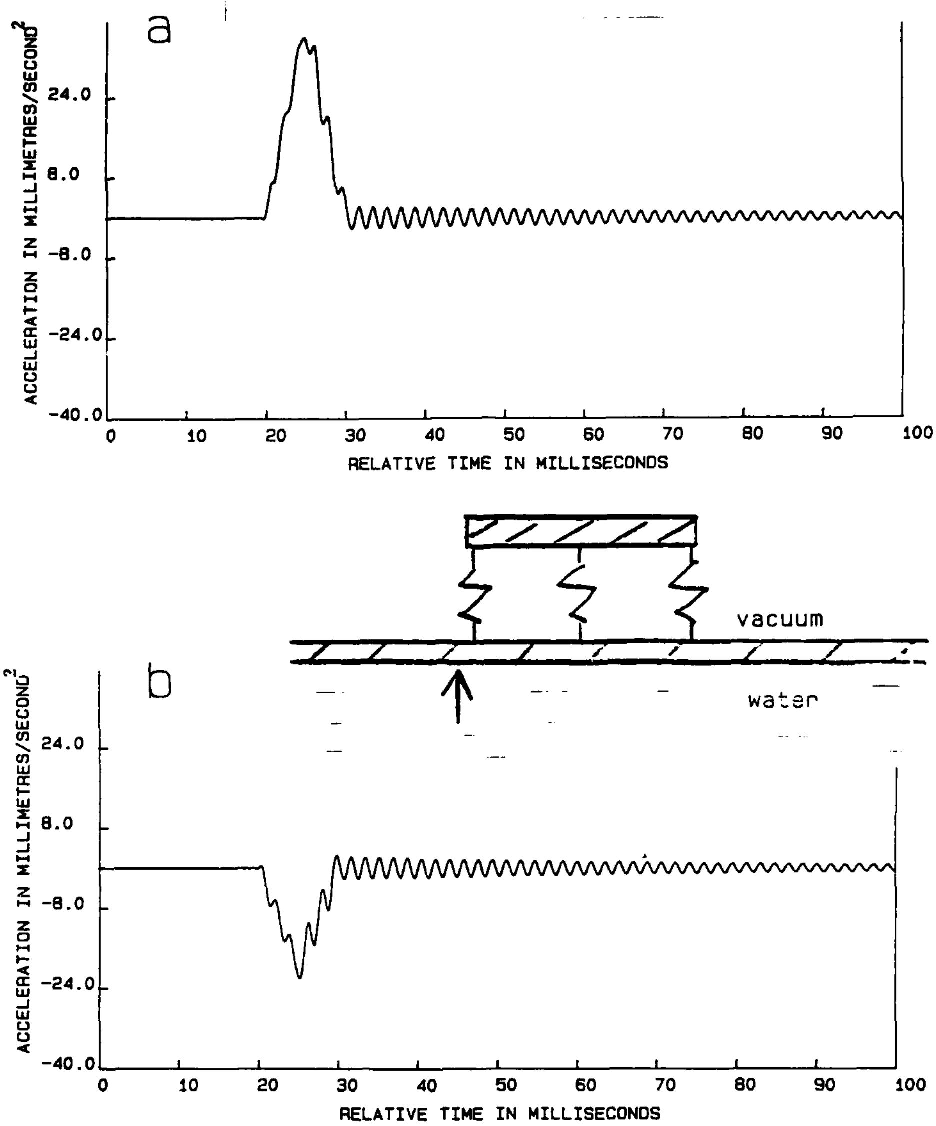


FIG. 4 Vertical acceleration response of isolation system excited by triangular pulse on for 10ms: (a) at near beam tip and (b) at far beam tip.

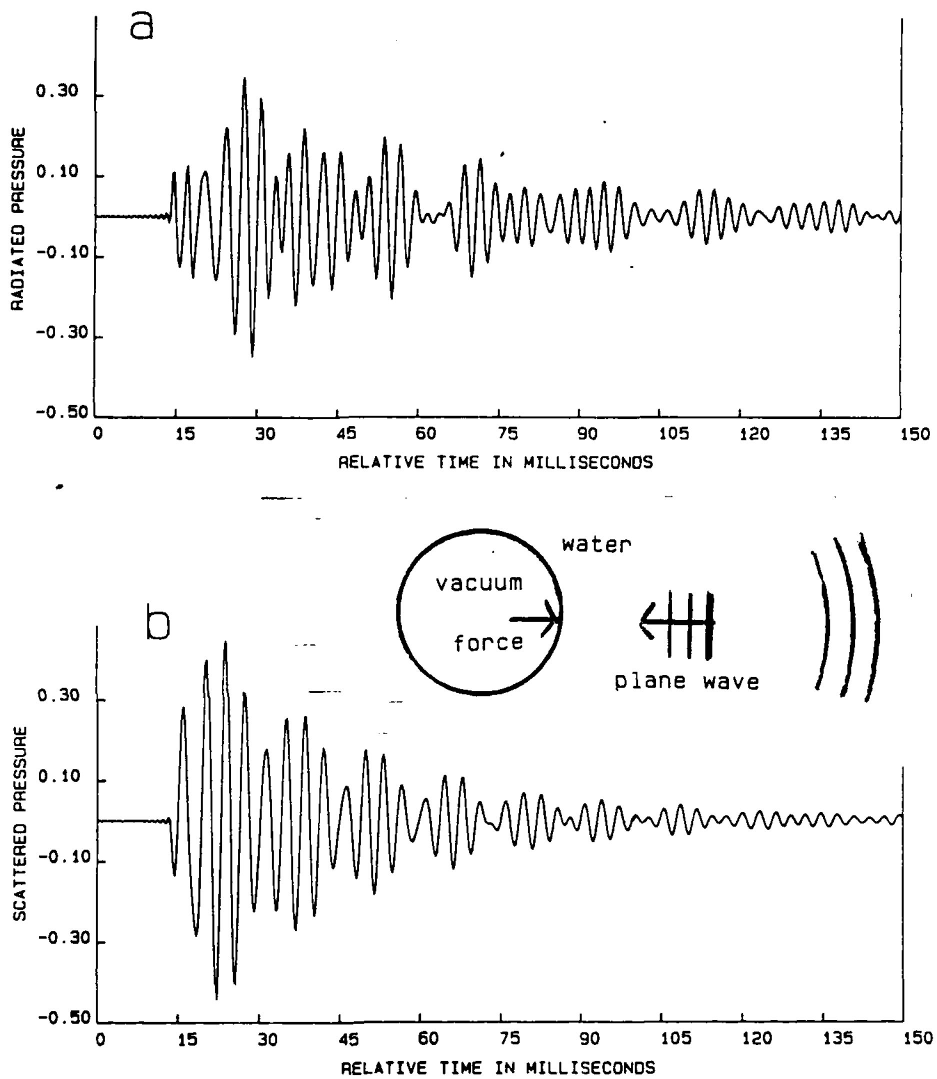


FIG. 5 Far-field pressure in pascals ref. 1m of submerged spherical shell excited by a 300 Hz sine wave on for 1 cycle: (a) force excitation (b) plane wave excitation.

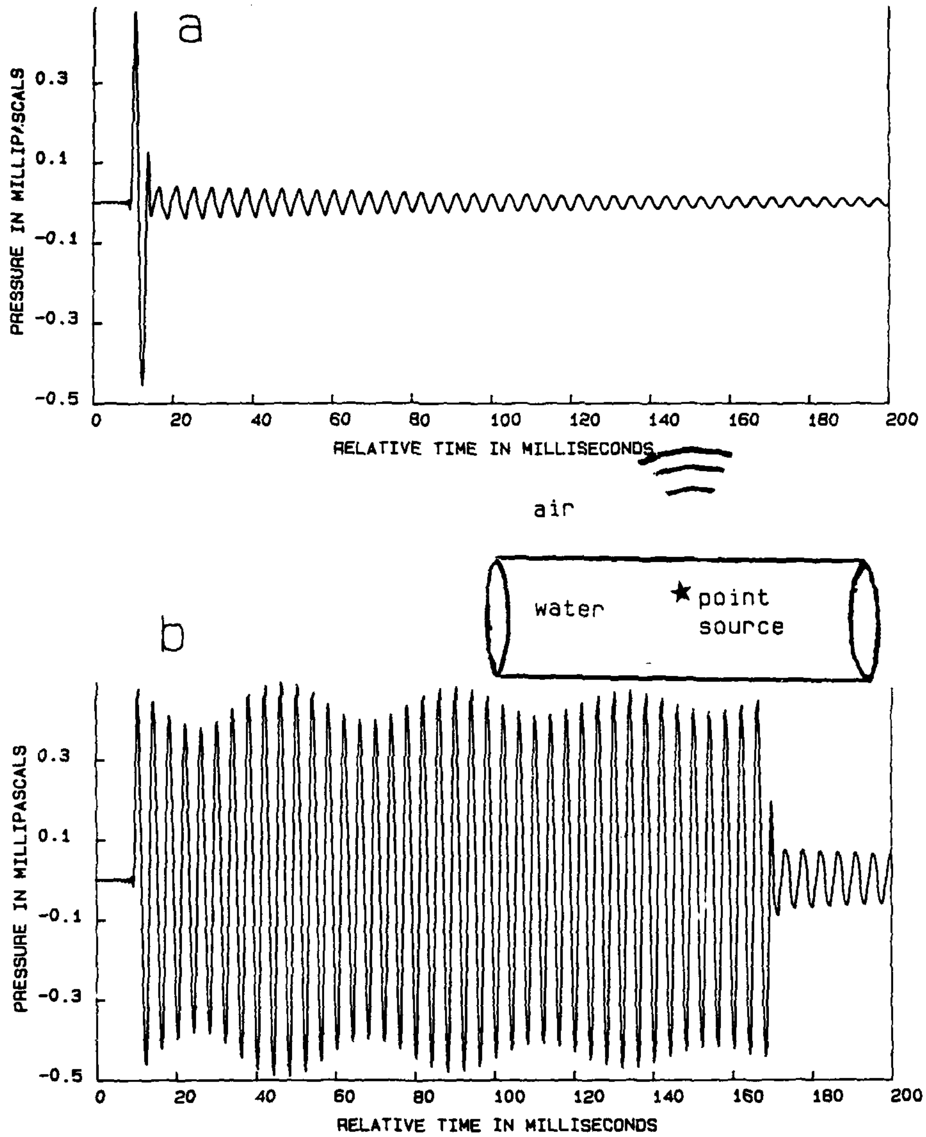


FIG. 6 Far-field airborne pressure in millipascals ref. 1m of water-filled pipe excited by 250Hz sine wave point source on for *(a) 1 cycle (b) 40 cycles*.

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Abstract
Theory behind a Fortran program ~~(not~~ listed) which facilitates computations of transient response is described. Discrete values of a time-harmonic spectrum are used, via quadratic interpolation and weighting by the transform of a selected time function, to provide samples for transient response calculations done by numerical evaluation of a Fourier integral using the fast Fourier transform (FFT) algorithm. Examples of motion, radiation and scattering transient responses of simple dynamical systems demonstrate that the program is a valuable research tool. This report is an update of previous work.